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Strings, Fivebranes and an Expanding Universe

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Abstract

It was recently shown that velocity-dependent forces between parallel fundamental strings moving apart in a $D-$dimensional spacetime implied an accelerating expanding universe in $D-1$-dimensional space-time. Exact solutions were obtained for the early time expansion in $D = 5, 6$. Here we show that this result also holds for fundamental strings in the background of a fivebrane, and argue that the feature of an accelerating universe would hold for more general p-brane-seeded models.

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In recent work [1,2], it was shown that velocity-dependent forces between parallel fundamental strings moving apart in a D-dimensional spacetime implied an accelerating expanding universe in $D-1$-dimensional spacetime. Exact solutions were obtained for the expansion rate for simple models in $D=5,6$ [2]. It was also noted in [2] that an accelerating universe can equally well arise for parallel p-branes, for arbitrary p, preserving half the spacetime supersymmetries and moving apart in the transverse space.

In this letter we investigate whether the feature of an accelerating universe holds for different types of p-branes. We first verify that the string result holds for parallel fivebranes moving apart and then consider strings moving in a fivebrane background. We find an explicit solution in the mean-field approximation for an expanding, accelerating universe consisting of strings moving in the fivebrane background.

For the fundamental string solution [3], we start with the combined action $S_{string} = I_D + S_2$, where

$$I_D = \frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-g}\left(R-\frac{1}{2}(\partial\phi)^2-\frac{1}{2\cdot 3!}e^{-\phi}H_3^2\right) \tag{1}$$

is the D-dimensional string low-energy effective spacetime action and

$$S_2 = -\frac{T_2}{2}\int d^2\zeta\left(\sqrt{-\gamma}\gamma^{\mu\nu}\partial_\mu X^M\partial_\nu X^N g_{MN}e^{\phi/2}+\epsilon^{\mu\nu}\partial_\mu X^M\partial_\nu X^N B_{MN}\right) \tag{2}$$

is the two-dimesional worldsheet sigma-model source action. Upper case latin leters denote indices of target space-time, lower case greek leters are worldvolume indices. g_{MN}, B_{MN} and ϕ are the spacetime canonical metric, antisymmetric tensor and dilaton, respectively, while $\gamma_{\mu\nu}$ is the worldsheet metric. $H_3 = dB_2$ and T_2 is the string tension equal to its mass/length. The string sigma-model action is related to the canonical metric via $g^s_{MN} = e^{\phi/2}g_{MN}$.

The fundamental string solution represents stationary macroscopic string parallel to the x^1 direction and is given by [3]

$$\begin{gathered} ds^2 = e^{3\phi/2}\left(-dt^2+(dx^1)^2\right)+e^{-\phi/2}\delta_{ij}dx^idx^j, \\ e^{-2\phi}\equiv h = 1+\frac{k_n}{r^n}, \qquad B_{01} = -h^{-1} \end{gathered} \tag{3}$$

where $n = D - 4$ (we assume $D > 4$), $r^2 = x^i x_i$ and the lower case latin indices run through the $D-2$-dimensional space transverse to the worldvolume ($i, j = 2, 3, \ldots, D-1$). The constant $k_n = \frac{2\kappa_D^2 T_2}{n\Omega_{n+1}}$, where Ω_{n+1} is the volume of the $n+1$-dimensional unit sphere.

The Lagrangian for a test fundamental string moving in the background of a parallel source string is then given by [1,4]

$$\mathcal{L}_2 = -mh^{-1}\left(\sqrt{1 - h\dot{x}^2} - 1\right), \tag{4}$$

where m is the mass of the string, $\dot{x}^2 = \dot{x}^i \dot{x}_i$ and the "$\cdot$" represents a time derivative. This Lagrangian does not depend explicitly on time, so that the total energy of the system is conserved. For $D = 5, 6$, the equation for conservation of energy can be integrated exactly [2].

For $D = 5$ a straightforward integration yields

$$\left(\frac{\rho+3}{\rho+1}\right)\ln\left(\sqrt{\frac{r}{a}} + \sqrt{\frac{r}{a}+1}\right) + \sqrt{\frac{r}{a}}\sqrt{\frac{r}{a}+1} = \sqrt{\frac{(\rho+2)^3}{\rho(\rho+1)^2}}\left(\frac{t-t_0}{k_1}\right), \tag{5}$$

where $\rho = \frac{E}{m}$, $a = \frac{\rho k_1}{\rho+2}$ and t_0 is a constant. For small r (or early time t) [2],

$$r \simeq \left(\frac{\rho+2}{\rho+1}\right)^2 \frac{t^2}{k_1}, \tag{6}$$

while for large r (or late time), $r \propto t$.

For $D = 6$, we obtain

$$\sqrt{r^2+a} + \sqrt{a}\left(\frac{\rho+2}{\rho+1}\right)\ln\left(\frac{r+\sqrt{r^2+a}-\sqrt{a}}{r+\sqrt{r^2+a}+\sqrt{a}}\right) = \sqrt{\frac{\rho(\rho+2)}{(\rho+1)^2}}(t-t_0), \tag{7}$$

where again $a = \frac{\rho k_2}{\rho+2}$ and t_0 is a constant. For small r,

$$r \simeq r_0 \exp\frac{t}{\sqrt{k_2}}, \tag{8}$$

while for large r we again find $r \propto t$. This late time behaviour is a general property of this kind of model and is valid for any D [1].

In [2] it was shown that some simple models of a string-seeded universe in $D = 5, 6$ have the same early time behaviour (6),(8) as corresponding source/test string systems. This suggests that the mean-field approximation of [1] provides a valid description of the early time expansion rate for these systems. Late time expansion rates for these models can also be obtained from conservation of energy and may represent a testable prediction (see [2] for further discussions).

The fundamental string in $D = 10$ is a solution of $3-$form version of $D = 10, N = 1$ supergravity. The dual $7-$form version of this theory [5] corresponds to supergravity coupled to the fivebrane $\sigma-$model [6,7]. The action for the supergravity fields $(g_{MN}, A_{MNPQRS}, \phi)$ is now

$$I_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2 \cdot 7!} e^{\phi} K_7^2 \right) \tag{9}$$

where $K_7 = dA_6$. I_{10} is the same action as I_D in (1) for $D = 10$ provided H_3 and K_7 are related via the duality transformation

$$K_7 = {}^*H_3 e^{-\phi}. \tag{10}$$

The fivebrane sigma-model action is given by [6,8]

$$\begin{aligned} S_6 = -T_6 \int d^6\zeta \Big(&\sqrt{-\gamma}\gamma^{\mu\nu}\partial_\mu X^M \partial_\nu X^N g_{MN} e^{-\phi/6} - 2\sqrt{-\gamma} + \\ &\frac{1}{6!}\epsilon^{\mu_1\mu_2\ldots\mu_6}\partial_{\mu_1}X^{M_1}\partial_{\mu_2}X^{M_2}\partial_{\mu_3}X^{M_3}\partial_{\mu_4}X^{M_4}\partial_{\mu_5}X^{M_5}\partial_{\mu_6}X^{M_6}A_{M_1M_2\ldots M_6}\Big) \end{aligned} \tag{11}$$

where T_6 is the fivebrane tension. The fivebrane sigma model metric is related to the canonical metric via $g^f_{MN} = e^{-\phi/6} g_{MN}$.

The fundamental fivebrane solution to the equations of motion of the combined action $S_{fivebrane} = I_{10} + S_6$ is given by

$$\begin{aligned} &ds^2 = e^{-\phi/2}\left(-dt^2 + (dx^1)^2\right) + e^{3\phi/2}\delta_{mn}dx^m dx^n, \\ &e^{2\phi} = 1 + \frac{\tilde{k}_2}{r^2}, \qquad A_{012345} = -e^{-2\phi} \end{aligned} \tag{12}$$

where $\tilde{k}_2 = \frac{\kappa_{10}^2 T_6}{\Omega_3}$, $m, n = 6, 7, 8, 9$ and r is the radial coordinate in the four-dimensional space transverse to the six-dimensional worldvolume of the fivebrane.

The Lagrangian for a test fivebrane moving in the background of a parallel source-fivebrane [9] can be easily obtained from (11), (12) and in term of the proper time of the test fivebrane takes the form

$$\mathcal{L}_6 = -me^{-2\phi}\left(\sqrt{1 - e^{2\phi}\dot{r}^2} - 1\right), \tag{13}$$

where m is the mass of the test fivebrane. Thus the Lagrangian (13) for this two-fivebrane system in $D = 10$ has exactly the same form as the Lagrangian for the test string moving in the source string background in $D = 6$. This fact is a consequence of string/fivebrane duality in $D = 10$ or even string/string duality in $D = 6$, once the fivebrane is reduced to a dual string in $D = 6$ [7]. Note that we could have obtained the identical result as above starting directly with the dual string in $D = 6$. So the early time dependence $r = r(t)$ (where r is the radial coordinate for the four-dimensional space transverse to the six-dimensional worldvolume) has the form $r \simeq r_0 \exp \frac{t}{\sqrt{\tilde{k}_2}}$ where r_0 is the initial distance between the fivebranes in this four-dimensional space. In the mean-field approximation, the position of the test-fivebrane represents the average fivebrane-fivebrane distance and hence the scale size of the universe (see [1,2]).

We now wish to investigate whether the above scenario holds for velocity-dependent forces between different branes. The most natural case to consider is that of a brane propagating in the background of a dual brane. In particular, we consider a test string moving in the background of a fivebrane. Suppose the fivebrane is oriented along $x^\alpha = \zeta^\alpha$ ($\alpha = 1, 2, \ldots, 5$). We assume that the test string lies either parallel or antiparallel to one of the fivebrane directions, say x^1. Viewed as a background for string propagation, the fivebrane is a nonsingular solution of the spacetime action I_{10} alone, without the need for a source term (since no singularity is present in the string frame). The metric and dilaton are then given by (12), with the three-form given by the duality transformation (10). Since, from (12), the only nonvanishing components of K are of the form $K_{012345m}$ where the directions $m = 6, 7, 8, 9$ are transverse to the fivebrane, by dualizing we obtain that the only nonzero components of $H_3 = dB_2$ are $H_{pqs}(r)$ where again $p, q, s = 6, 7, 8, 9$. Thus the only nonvanishing components of B_{MN} occur when $M, N = 6, 7, 8, 9$. It then follows that the Wess-Zumino-Witten term in the action (2) for the test string in the fivebrane background $\epsilon^{\mu\nu}\partial_\mu X^M \partial_\nu X^N B_{MN}$ vanishes. Replacing the fields of the fivebrane from (12) into (2) we obtain the Lagrangian [9]

$$\mathcal{L} = -m\sqrt{1 - e^{2\phi}\dot{r}^2} \tag{14}$$

where m is the mass of the test string and $e^{2\phi}$ is given by (12). The Hamiltonian of this test string/source fivebrane system also does not depend on time and thus represents the conserved energy

$$E \equiv H = \frac{m}{\sqrt{1 - e^{2\phi}\dot{r}^2}}. \tag{15}$$

Integrating (15) over time we obtain

$$\sqrt{r^2 + \tilde{k}_2} - \sqrt{\tilde{k}_2}\ln\frac{\sqrt{r^2 + \tilde{k}_2} + \sqrt{\tilde{k}_2}}{r} = \sqrt{\alpha}t + const, \tag{16}$$

where $\alpha = 1 - \frac{m^2}{E^2}$.

For early times ($r << \sqrt{\tilde{k}_2}$) we obtain from (16)

$$r \simeq r_0 \exp\left(\sqrt{\frac{\alpha}{\tilde{k}_2}}t\right) \tag{17}$$

i.e. the same type of early time dependence as for parallel strings in $D = 6$. Once again, for late times we obtain $r \propto t$ as a general feature of this type of model [1,2]. We would expect this feature to persist for any dual pair of p-brane, in particular for fivebranes propagating in the background of a string.

Since similar results seem to hold for branes propagating in either identical or dual brane backgrounds, this strongly suggests that the type of velocity-dependent forces present in the above cases are generic to branes and would lead to an accelerating expanding universe for various different types of branes simultaneously propagating in the same background. This is also supported by the compositeness feature of brane solutions, which allows for the construction of arbitrary brane solution from fundamental brane building blocks (of which the fundamental string and fivebrane are examples) [10,11]. Another interesting question is whether these scenarios hold in the context of general relativity, independent of string theory. A possible extension of these results is to consider different orientations of the various branes, in which case the zero-force condition no longer holds in the static limit. Finally, it is worthwhile to go beyond the mean-field approximation and investigate the many-body problem directly (see [2] for some simplified models), also taking into account quantum interactions.

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